INDIANA UNIVERSITY

BORKENSTEIN COURSE

Center for Studies of Law in Action

Practice Mathematical Problems Relevant to Breath Alcohol Testing Programs

by
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Mathematical and Statistical Applications For Breath Alcohol Test Programs

Practice Problems

(Use the attached standard normal, t-tables and equations)

1. An individual, male aged 35 weighing 180 lbs., begins drinking at 7 pm and stops at 11 pm. He is arrested for DUI at 1 am the next morning and is administered a breath test at 2 am. The duplicate breath alcohol results are 0.135 and 0.143 g/210L. Determine the estimated number of 12 fluid ounce beers (assume 4% alcohol by volume) that the individual would have consumed along with a ±25% interval of uncertainty. Assume Widmark's $\rho = 0.70$ and $\beta = 0.017$ g/100ml/hr. Use Equation 1.

2. An individual, female aged 30 and weighing 125 lbs., begins drinking at 9 pm and continues until 12 midnight. She consumes eight drinks, each containing one fluid ounce of 80 proof vodka. Estimate what her blood alcohol concentration would be at 2 am the next morning along with a ±40% interval of uncertainty. Assume Widmark's $\rho=0.60$ and $\beta = 0.015$ g/100ml/hr. Use Equation 1.

3. An individual, male aged 25 weighing 160 lbs, begins drinking at 8pm and stops at 1 am. During this time he consumes nine 12 fluid ounce beers (assume 4% by volume) and five glasses of vodka each containing one fluid ounce of 80 proof. Estimate what his blood alcohol concentration would be at 2 am along with a ±25% interval of uncertainty. Assume Widmark's $\rho=0.72$ and $\beta = 0.018$ g/100ml/hr. Use Equation 1.

4. An individual, male aged 45 weighing 190 lbs., begins drinking at 6 pm and stops at 10 pm. He is arrested for DUI at midnight and is administered a breath test at 1 am. The duplicate breath alcohol results are 0.092 and 0.085 g/210L. Determine the estimated number of 12 fluid ounce beers (assume 4% alcohol by volume) that the individual would have consumed along with a ±25% interval of uncertainty. Compute also the 2 standard deviation uncertainty interval using Widmark's uncertainty equation. Assume Widmark's $\rho = 0.70$ and $\beta = 0.014$ g/100ml/hr. Use Equations 1 and 2.

5. An individual, male aged 38, weighing 170 lbs and height of 5 feet 10 inches, begins drinking at 7pm and stops at 11pm. He is arrested for DUI at midnight and is administered a breath test at 1am with results of 0.145 and 0.152 g/210L. Determine the estimated number of 12 fluid ounce beers (assume 4% alcohol by volume) that he consumed using both Widmark's equation and the equation of Watson, et.al., for

total body water (TBW). Assume $\rho = 0.72$ and $\beta = 0.018$ g/100ml/hr. Determine also a ±25% interval of uncertainty. Use equations 1, 15 and 16.

6. Assume that an individual provides duplicate breath samples resulting in 0.097 and 0.106 g/210L. When a new simulator solution (reference value = 0.083 g/210L) was installed on the instrument about one month earlier the first three results were: 0.081, 0.083 and 0.080 g/210L. Determine the 99% confidence interval for the within-subject population mean breath alcohol concentration. Find the standard deviation from: SD = 0.0305B+0.0026 and use t=2.57. Use Equations 3 and 4.

7. An individual provides duplicate breath samples resulting in 0.088 and 0.091 g/210L in an instrument that has a bias of +3.0%. The simulator reference value of 0.082 g/210L has a combined uncertainty (standard deviation) of 0.0014 g/210L as determined from n=10 measurements on the gas chromatograph. Assume the combined biological and analytical components of variation are found from: SD = 0.0305B+0.0026. Correct the subject's mean breath alcohol concentration and find the 99% confidence interval by combining all sources of uncertainty. Use t=2.57 and equations 3,4 and 14.

8. Assume that your Toxicology Lab receives a CRM from Cerilliant with an unbiased reference value of 0.1025 g/100ml and a combined uncertainty of 0.0012 g/100ml. The Toxicology Lab performs replicate measurements (n=5) on this CRM and obtains a maximum bias of 0.003 g/100ml and a standard deviation of 0.0007 g/100ml on a mean value of 0.0995 g/100ml. The Toxicology Lab does not correct for bias. Next, the Toxicology Lab prepares simulator solutions and performs replicate measurements (n=15) obtaining a mean of 0.1005 g/100ml with a standard deviation of 0.0006 g/100ml. When this solution is heated in a simulator to 34^{0}C, a breath test instrument obtained a mean value from replicate measurements (n=10) of 0.0815 g/210L with a standard deviation of 0.0010 g/210L during a calibration procedure. Assume a partition coefficient of 1.23 with an uncertainty of 0.0124. Determine the bias in the breath test instrument along with a combined uncertainty.

9. Assume that you are preparing an ethanol reference standard. Your preparation function is:

$$C = \frac{mP}{V}$$

where: C = concentration of ethanol (g/100ml)
m = mass measure of ethanol (g)
P = purity as a mass fraction
V = volume (L)

You carefully weigh out one gram of ethanol which has a combined uncertainty (standard deviation) estimate of 0.005g. This combined uncertainty includes traceability, replication and scale resolution. The purity of the ethanol is reported as 0.99 ± 0.001. Assume the uniform distribution for this uncertainty estimate. You mix the ethanol with water to a total volume of one liter. The flask is specified as 1.00 ± 0.02 L. Assume the uniform distribution for this uncertainty estimate. Determine the combined uncertainty in the reference concentration C.

10. Assume that during a one month period for a particular jurisdiction there were 458 women and 1,860 men arrested for DUI. Amongst these, 64 women and 298 men refused to submit to a breath test. Construct a 95% confidence interval for the proportion that refuse the test for each gender. Construct also a 95% confidence interval for the difference between the two proportions. Finally, construct a two-way contingency table that uses the χ^2 test to evaluate for independence between gender and refusal rate. Assume Z=1.96 for the confidence intervals. Would you conclude there is a significant difference between the two refusal rates? Use equations 17, 18 and 19.

11. Assume that an individual provides duplicate breath samples resulting in 0.092 and 0.098 g/210L. When a new simulator solution (reference value = 0.082 g/210L determined from 30 measurements with a standard deviation of 0.0010 g/210L) was installed on the instrument about two weeks earlier the first five results were: 0.086, 0.084, 0.083, 0.086 and 0.083 g/210L. Determine the 99% confidence interval for the within-subject population mean breath alcohol concentration. Include the uncertainty in the breath test instrument measurement of the simulator standard and the gas chromatography measurement of the simulator solution. Find the standard deviation for the subject's results from: SD = 0.0305B+0.0026 and use t=2.57.
Use Equations 3, 4 and 5.

12. An individual provides two breath samples that result in 0.084 and 0.087 g/210L. Determine the probability that individual's true mean breath alcohol concentration exceeds 0.080 g/210L. Assume that there is no systematic error and estimate the standard deviation for individual breath alcohol results from: SD = 0.0305B+0.0026.
Use Equation 6.

13. An individual provides duplicate breath alcohol results of 0.088 and 0.095 g/210L. Assume the standard deviation associated with these individual analyses is 0.0053 g/210L. How long would you have to go back in time prior to the analyses in order to perform duplicate analyses and assume that you would be able to measure a difference in the sample means? Assume β = 0.015 g/210L/hr. and that you need a critical difference of: $\delta_{cr} = 2.77(S_{\bar{Y}})$

14. You are evaluating the accuracy of a breath test instrument and have performed n=10 simulator measurements and obtained a mean of 0.099 g/210L and a standard deviation of 0.0012 g/210L. The simulator solution you used was tested by gas chromatography and yielded a mean of 0.103g/210L with a standard deviation of 0.001 g/210L in m=30 measurements. Construct a 99% confidence interval for the difference between the breath test instrument results and the reference value (the systematic error). Express the confidence interval as percentages also. Use Equation 7.

15. An article in the literature reported the study of acutely ill diabetic patients in a condition known as diabetic keto-acidosis (Owen, O.E., Trapp, V.E., Skutches, C.L., Mozzoli, M.A., Hoeldtke, R.D., Boden, G. and Reichard, G.A., "Acetone Metabolism During Diabetic Ketoacidosis", Diabetes, Vol.31, 1982, pp. 242-248). The maximum reported plasma acetone concentration observed was 8.91 mM (milliMoles) per liter. Assume the blood/breath partition coefficient for acetone is 300 and that 58 g of acetone equals 1 Mole. If an infrared breath test instrument requires 642 μg/L of acetone in the breath to yield 0.01 g/210L ethanol equivalent, what result would we expect from the extreme patient reported?

16. Assume that the number of tests performed per day on a particular breath test instrument follow the Poisson distribution with mean $\lambda = 40$. If 55 or more tests are performed per day then a second instrument will be necessary to install. Determine the probability that 55 or more tests will be performed on a particular day. Hint: Use the normal approximation to the Poisson where both the expected value (mean) and variance is λ.

17. A study ("Ethanol Metabolism in Men and Women", Journal of Studies on Alcohol, Vol.48, 1987, pp. 380-387) compared the elimination rates of ethanol in men and women. The study found (table 1 of the study) the mean elimination rate and for men was $\beta_M = 17.24$ mg/dl/hr (n=75) while that of women was $\beta_F = 20.77$ mg/dl/hr (m=59). The standard errors of the mean ($SE_{\bar{Y}} = S_{\bar{Y}} = \frac{S_Y}{\sqrt{n}}$) were also noted to be $SE_M = 0.40$ and $SE_F = 0.55$ for the men and women respectively. Use a two sample t-test (two-tailed) assuming equal variances to test the hypotheses: H_0: $\beta_M = \beta_F$ H_1: $\beta_M \neq \beta_F$ at $\alpha=0.01$. Use equation 8.

18. You are interested in determining whether there is a significant difference between simultaneously collected within-subject blood and breath alcohol samples. Since we must employ the same units for both measurements, assume that the BrAC has units of g/100ml. You are only concerned if there is a systematic difference (δ) of 0.005 g/100ml. Assume the standard deviation of the differences between the paired measurements is 0.007 g/100ml. Determine the sample size you would need in order to detect a difference of

0.005 g/100ml as being significant at a level of $\alpha=0.05$ with a power level of 0.80. Use Equation 9.

19. Some have argued that we should not report the final BrAC at the end of exhalation but rather we should report the average value over the exhalation time. Assume an individual exhales for 10 seconds and provides a breath alcohol exhalation curve that is modeled by:

$$B_t = 0.15(1-e^{-2t}) + 0.003t \qquad Eq.1$$

Find the average value for this model at t=5 seconds and again at t=10 seconds. Determine also the function that models the average value as a function of time. Is the average value a constant percentage of the function in equation 1? (You will need to integrate equation 1 over time t and then divide this by t)

20. "Alcohol free" beers claim to contain less than 0.5% alcohol by volume. You want to test this claim so you place one 12 fluid ounce "alcohol free" beer in a simulator, heat it to 34^0 C and perform five measurements with a properly calibrated breath test instrument. You obtain the following results: 0.285, 0.281, 0.278, 0.282, 0.274. Test the null hypothesis H_0: $\mu_0 < 0.5\%$ against H_1: $\mu_0 \exists 0.5\%$ at the $\alpha=0.05$ level. Use Equations 10 and 11.

21. One individual provided n=10 replicate breath samples into a breath test instrument within 12 minutes and obtained the following results: 0.079, 0.079, 0.078, 0.078, 0.078, 0.074, 0.075, 0.075, 0.075, 0.076. Calculate the mean, standard deviation and coefficient of variation for the results based on two digit truncated results and based on all three digits. Why do we obtain different results? Should two or three digits be employed? Use Equations 12 and 13.

22. An individual provides only one breath sample resulting in 0.116 g/210L and then refuses the second. Compute a 99% confidence interval for the population mean breath alcohol concentration using the following equation to estimate the standard deviation: SD = 0.0305B+0.0026. Let t=2.57. Use Equations 3.

23. The BAC Datamaster uses the following equation to compute the breath alcohol concentration from the dc voltage generated from the detector:

$$X_a = -1.3\left[ln(1-\frac{V}{5})\right] \qquad Eq.1$$

The microprocessor, however, is not capable of handling the logarithmic function so an approximation is used derived from a Taylor's series expansion. Only the first three terms of the infinite series are used which is seen below.

$$f(x) = -1.3\left[-x - \frac{x^2}{2} - \frac{x^3}{3}\right] \qquad \textit{Eq. 2} \qquad \text{where:} \quad x = v/5$$

Compare the accuracy of using the approximation in equation 2 with the direct calculation in equation 1 when the voltage (v) = 0.40. Compare the two results when the voltage is v=1.1. Which of the two methods is always lower?

24. You obtain a blood sample from a vehicular homicide case and have it analyzed twice in your toxicology laboratory with results: 0.084 and 0.086 g/100ml. The defendant gets the sample also and has it analyzed with the following results: 0.083, 0.084, 0.083, 0.083, 0.085 g/100ml. You send it off to a third lab for re-analysis. They do two runs with duplicates each and obtain: 0.084, 0.086, 0.085, 0.088 g/100ml. Assume that none of the methods employed had any systematic error. What is the estimate of the individual's "true" blood alcohol concentration?

25. Assume that you calibrate a breath test instrument with a simulator using a mercury thermometer that is reading 0.3^0 C too low. You calibrate the instrument with a simulator solution prepared to produce a vapor alcohol concentration of 0.082 g/210L, as determined by the Toxicology Laboratory. Later, an arrested subject provides breath samples resulting in 0.125 and 0.132 g/210L. What would be the subject's corrected mean BrAC? Assume that a one degree centigrade change in temperature results in a 6.5% change in headspace alcohol concentration.

26. Assume that the elimination rate of "mouth alcohol" for a person with alcohol in their system is defined according to the following differential equation:

$$\frac{dB}{dt} = -k(B - B_0)$$

where: B = breath alcohol concentration at time t
B_0 = mean end-expiratory breath alcohol concentration
t = time

Show that solving this differential equation yields the following "mouth alcohol" elimination model with three parameters: $B = C_0 e^{-kt} + B_0$
Assume that non-linear regression of "mouth alcohol" elimination data yields the following parameter estimates:
$C_0 = 0.85\, g/210L,\ k = 0.5\, min^{-1},\ B_0 = 0.150\, g/210L$. If 0.165 g/210L represents three standard deviations above the mean end-expiratory breath alcohol concentration, at what time would the "mouth alcohol" decrease to this concentration?

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